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Edge Length Interpolation

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Problem

- Input: set of meshes with same triangulation
  - Interpolate edge lengths for target mesh
  - Embed target mesh in $\mathbb{R}^3$

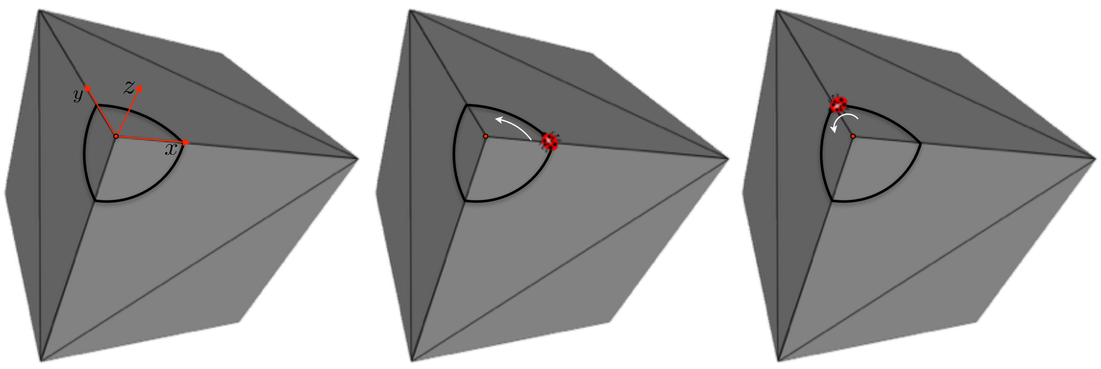
Why Interpolate Edge Lengths?

- They are shape invariant to rotation and translation.
- Vectors of edge lengths can be seen as points in a Euclidean shape space so we can easily perform statistical shape analysis.

Embedding Method

- Solve for the dihedral angles
- Reconstruct vertices from dihedral angles and edge lengths

Solve for the dihedral angles



- Vertex figure: Intersection of polyhedron with infinitesimal sphere at v.
- The ladybug's walk is a series of Z and X rotations in her local coordinate system.
- Minimizing E_v gives optimal dihedral angles.

$$E_v = ||Z_0 X_0 Z_1 X_1 \dots Z_k X_k - I||_F^2$$

Reconstruct vertices from dihedral angles and edge lengths

- Matrix $M_{v,u}$ transforms local coordinate system of v to local coordinate system of u; known from edge lengths and dihedrals
- Compute the orientation of every local coordinate frame G_v in a single global coordinate system by solving

$$G_v M_{v,u} - G_u = 0$$

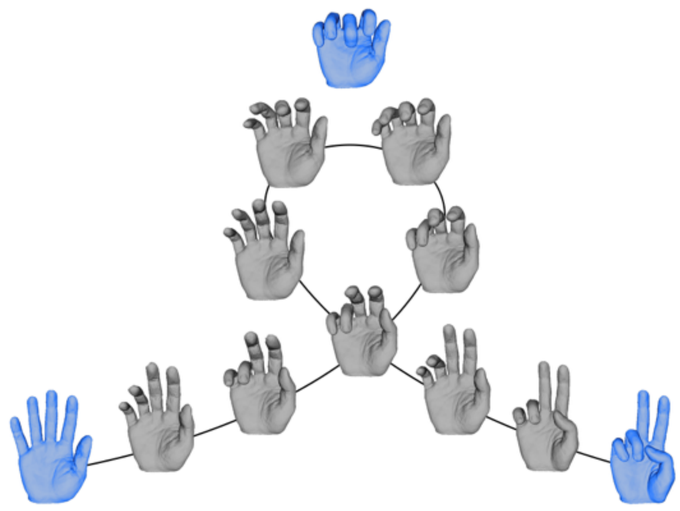
- For every edge u, v let u_v be position of u in the coordinate system of v.
- Global positions of u and v satisfy.

$$v + G_v u_v - u = 0$$

Results

Four edge-length interpolation steps between straight and bent arm.

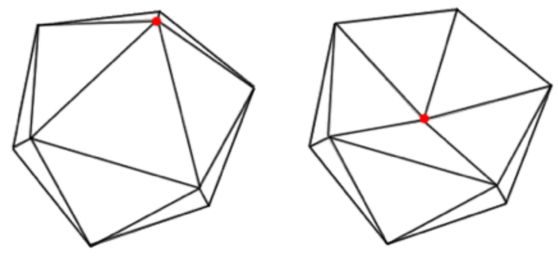
Bilinearly interpolating the edge lengths of the blue centaurs on the four corners produces the centaurs in the center.



The gray poses of the hands are interpolated from the three blue poses by interpolating their edge lengths.

Limitations

- Computation is slow, due to solving a high-dimensional non-linear system.
- Where a creases appear or disappear we see jitter
- Self-intersections are not allowed
- Usually there are multiple correct embeddings as shown in icosohedron below



For more information, please contact Carlos Rojas at crojas@ucdavis.edu

71.11 cm (W) x 91.43 cm (H)